International Journal of Thermophysics, Vol. 17, No. 2, 1996

Use of a Radiation Diffusion Model for Determination of Optical and Thermal Radiative Properties of Anisotropic Silica Fiber Thermal Insulation

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Received July 10, 1995

A radiation diffusion model is used to describe radiation transfer in highly scattering anisotropic semitransparent materials. The transmissivities of the sample disks of silica glass fiber thermal insulation of different thicknesses and orientations were measured. The samples were cut from the same insulation slab. The axis of material structure symmetry was parallel to the axis of samples in one case and perpendicular to it in the other case. The effective absorption coefficient and radiation diffusion coefficient were obtained for three wavelengths of a probing He-Ne laser, namely, 0.63, 1.15, and 3.39 μm. The spectral hemispherical absorptivity and bihemisperical reflectivity as a function of the thickness of the plane layer are calculated and presented.

KEY WORDS: glass fiber; optical properties; radiation properties; thermal insulation.

1. INTRODUCTION

In recent years, lightweight insulation based on refractory oxide fibers has been used in space engineering, aviation, and other fields of high-temperature technology. The fibrous oxide thermal insulations are semitransparent materials that have a broad wavelength range of low absorption and high scattering of thermal radiation. For example, this range is from about 0.3 to 3.5 μm for silica glass-fiber thermal insulation. The absorption coefficient of silica glass is less than 0.03 cm^{-1} in this range at room temperature. It is important to investigate the radiation energy transfer in this wavelength range at temperatures of 800–1500 K.

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0195-928X/96/0100-0515$09.50/0 © 1996 Plenum Publishing Corporation

The customarily used approaches are based on the use of the radiation transfer equation (RTE) of radiative thermal conductivity approximation. However, the application of these approaches faces some difficulties [1–3].

As shown in Ref. 2, application of the radiative thermal conductivity approximation may cause large errors in the calculation of the temperature field, expecially for transient conditions. This is due to disturbance of a local radiant equilibrium in highly scattering and weakly absorbing materials like glass-fiber thermal insulation.

The application of RTE, in common with the Mie theory for calculating optical properties of fiber thermal insulation, may lead to errors due to the assumption of independent scattering, which takes place when the distance between scatterers is large compared to their size and wavelength. Besides this, it is necessary to take into account the real distribution of fibers in thermal insulation by diameter and direction. There are also some other disadvantages of calculation of optical properties based on the Mie theory: the use of literature data on the absorption coefficient of fiber material and the assumption of the fibers as infinite homogeneous cylindrical scatterers. The presence of impurities and technological additives, even if present in a fiber material in a small and uncontrolled amount, can lead to a very great increase in the absorption coefficient in the wavelength range of low absorption. The real fibers in thermal insulation bend and cohere to one another at the junctions (usually there are three or four junctions per fiber), so they cannot be considered infinite cylindrical scatterers.

Experimental determination of the optical properties of fibrous thermal insulation as a result of the solution of some inverse radiation transfer problems in the frame of the use of RTE in its strict form has not been obtained to date [3].

The diffusion model is the most acceptable for describing radiation transfer in highly scattering low-absorbing materials in the diffusion limit conditions

$$kD \ll 1; \qquad D/l \ll 1 \tag{1}$$

where k is the effective absorption coefficient [4], D is the radiation diffusion coefficient, and l is the distance from a point in the interior of the material to any point on the boundary. In this case, the radiation diffusion model is not considered as a radiation transfer equation approximation which is applicable for rarefied media. This model is proposed as an original approach, which is based on the macroscopic law of monochromatic radiation energy conservation

$$\operatorname{div} \mathbf{q} = -kU \tag{2}$$

and the phenomenological Fick law for diffusion of radiation

$$\mathbf{q} = -D\,\nabla U \qquad (3)$$

The values $\mathbf{q}$ and U in Eqs. (2) and (3) are macroscopic (i.e., averaged over an elemental volume which is infinitely small physically and contains a large enough number of scatterers) spectral radiation flux and volumetric radiation energy density. For simplicity, the subscript λ expressing the spectral nature of the relevant quantities has been omitted.

Equation (2) can be considered to be an expression that defines the effective absorption coefficient in the radiation diffusion model. As the left-hand side of this equation is scalar, the effective absorption coefficient defined this way does not depend on the direction, whereas the absorption coefficient used in RTE may have the angular dependence for scattering media of anisotropic struture.

Unlike k, in accordance with Eq. (3), the radiation diffusion coefficient D of anisotropic media is a tensor. In this case there is full analogy with the thermal conductivity tensor in the Fourier law.

A significant advantage of the diffusion model over RTE is that it operates with a lesser number of parameters. For example, for an isotropic medium, it needs only two parameters, the effective absorption coefficient k and the radiation diffusion coefficient D, while for use of the radiation transfer equation the absorption coefficient, the scattering coefficient, and the angular dependence of the scattering phase function must be known.

This advantage is more striking when an anisotropic medium with axially symmetrical structure is considered. In this case, the diffusion model operates with three parameters only: k, D_x, and D_r, where D_x and D_r are the axial and radial components of the tensor of radiation diffusion reduced to the principal axes. If RTE is used, then the angular dependencies of the absorption and scattering coefficients and bidirectional phase function of scattering must be known.

It must be noted once again that application of the radiation diffusion model is limited by the conditions expressed by Eq. (1), i.e., the model can be applied to low-absorbing and high-scattering material, and the thickness of the layer must be large enough in comparison with the mean free path of radiation. Usually, high-temperature glass fiber thermal insulations fully meet these demands.

For an anisotropic medium, the value of D in the first inequality of the conditions in Eq. (1) must be the maximum principal value of the tensor of the radiation diffusion, and in the second inequality in Eq. (1) it must be the value of the tensor of the radiation diffusion in the direction from a considered point inside the medium to any point on the boundary.

The reason for the anisotropy of the optical (also thermophysical and mechanical) properties of thermal insulation is the possible preferential orientation of fibers in some directions. The majority of studies published in the literature does not take into account the possible influence of this factor. The studies by Lee [5–7] are an exception. In these studies, the calculations of optical properties of fibrous media are carried out using the Mie theory for certain fiber orientations in the polar direction and random orientation in the azimuthal directions.

2. DIFFUSION OF RADIATION IN A CYLINDRICAL SAMPLE

The alternative approach described below is based on experimental determination of optical properties k, D_x, and D_r as a result of the solution of the inverse radiation diffusion problem in cylindrical samples. In this case, the radiation diffusion equation and its boundary conditions are

$$D_x \frac{\partial^2 U}{\partial x^2} + \frac{D_r}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial U}{\partial \rho} - kU = 0 \tag{4}$$

$$-D_x U_x' + \psi_1 U = 4\psi_1 n^2 q_0(\rho); \qquad x = 0 \tag{5}$$

$$-D_x U_x' - \psi_2 U = 0; \qquad x = L \tag{6}$$

$$-D_r U_r' - \psi_s U = 0; \qquad \rho = \rho_0 \tag{7}$$

where x and ρ are axial and radial coordinates; L and ρ_0 are the length and radius of the cylinder; k and n are the effective absorption coefficient and refractive index, which are related to the absorption coefficient k_0 and the refractive index n_0 of the fiber material and porosity Π by

$$k = \frac{k_0 n_0^2 (1 - \Pi)}{n_0^2 (1 - \Pi) + \Pi}; \qquad n = [n_0^2 (1 - \Pi) + \Pi]^{1/2} \tag{8}$$

$\psi_{1(2)} = 0.5(1 - r_{h1(2)})/(1 + r_{h1(2)})$; $\psi_s = 0.5(1 - r_{hs})/(1 + r_{hs})$; r_{h1}, r_{h2}, and r_{hs} are the internal bihemisperical reflection coefficients of face $1 (x = 0)$, face $2 (x = \mathrm{L})$, and the side surface; and $q_0(\rho)$ is the external monochromatic flux of radiation impinging diffusively on the sample. For an uniform energy distribution, $q_0(\rho) = Q_0/\pi\rho_b^2$ at $\rho \leqslant \rho_b$ and $q_0(\rho) = 0$ at $\rho > \rho_b$ (Q_0 is the total incident on the face 1 energy rate). Equations (4)–(7) are written in cylindrical coordinates, where the axis of symmetry of the material structure is coincided with the axis of the cylinder.

By analogy with the cylindrical sample of istropic structure [3, 8], the solution is obtained by the Fourier method,

$$U(\rho, x) = \frac{4n^2 Q_0}{\pi \rho_0^2} \sum_{j=1}^{\infty} \frac{A_j}{B_j} J_0(\mu_j \rho/\rho_0)((1 + D_x \zeta_j/\psi_2) \exp(-\zeta_j x) - (1 - D_x \zeta_j/\psi_2) \exp(\zeta_j (x - 2L))) \quad (9)$$

$$A_j = \frac{2\rho_0}{\rho_b \mu_j} \frac{J_1(\mu_j \rho_b/\rho_0)}{[J_0^2(\mu_j) + J_1^2(\mu_j)]} \quad (10)$$

$$B_j = (1 + D_x \zeta_j/\psi_1)(1 + D_x \zeta_j/\psi_2) - (1 - D_x \zeta_j/\psi_1)(1 - D_x \zeta_j/\psi_2) \exp(-2\zeta_j L) \quad (11)$$

$\zeta_j = (k/D_x + \mu_j^2/\rho_0^2)^{1/2}$, $a = D_x/D_r$ is the anisotropy factor, and μ_j is the root of the characteristic equation

$$J_0(\mu) - \mu(D_r/\psi_s \rho_0) J_1(\mu) = 0 \quad (12)$$

where $J_0(\mu)$ and $J_1(\mu)$ are the zeroth- and first-order Bessel functions.

3. THERMAL RADIATIVE PROPERTIES OF A CYLINDRICAL SAMPLE

Let us determine the bihemispherical reflectivity R_h and transmissivity P_h of a cylindrical sample as the ratio of the radiation energy rate coming from face $1(Q_r)$ and face $2(Q_t)$ according to the incident external radiation energy rate Q_0 impinging isotropically on face 1. In this case,

$$R_h = \frac{Q_r}{Q_0} = 1 - \frac{Q(x=0)}{Q_0} = 1 + \frac{2\pi D_x}{Q_0} \int_0^{\rho_0} \frac{\partial U(x=0, \rho)}{\partial x} \rho \, d\rho \quad (13)$$

$$P_h = \frac{Q_t}{Q_0} = \frac{Q(x=L)}{Q_0} = -\frac{2\pi D_x}{Q_0} \int_0^{\rho_0} \frac{\partial U(x=L, \rho)}{\partial x} \rho \, d\rho \quad (14)$$

Using the boundary conditions given by Eqs. (5) and (6),

$$R_h = 1 + \frac{2\pi \psi_1}{Q_0} \int_0^{\rho_0} U(x=0, \rho) \rho \, d\rho - 4n^2 \psi_1 \quad (15)$$

$$P_h = \frac{2\pi \psi_2}{Q_0} \int_0^{\rho_0} U(x=L, \rho) \rho \, d\rho \quad (16)$$

The side surface bihemispherical transmissivity P_{hs} may be defined as the ratio of the radiation energy rate coming from the side surface of cylinder Q_{ts} to Q_0,

$$P_{hs} = \frac{Q_{ts}}{Q_0} = -\frac{2\pi\rho_0 D_r}{Q_0} \int_0^L \frac{\partial U(x, \rho = \rho_0)}{\partial \rho} dx \tag{17}$$

or, using the boundary condition given by Eq. (7),

$$P_{hs} = \frac{2\pi\rho_0 \psi_s}{Q_0} \int_0^L U(x, \rho = \rho_0)\, dx \tag{18}$$

The hemispherical absorptivity E_h may be defined from the energy conservation law as $E_h = 1 - R_h - P_h - P_{hs}$. Using in Eqs. (15), (16), and (18), the expressions for $U(x = 0, \rho)$, $U(x = L, \rho)$, and $U(x, \rho_0)$ obtained from Eq. (9), after integration we obtain the expressions for bihemispherical thermal radiative properties of the cylindrical sample of anisotropic axially symmetrical structure:

$$R_h = 1 - 8n^2 D_x \sum_{j=1}^{\infty} \frac{A_j J_1(\mu_j)\, \zeta_j}{B_j \mu_j} \left[1 + \frac{D_x \zeta_j}{\psi_2} + \left(1 - \frac{D_x \zeta_j}{\psi_2}\right) \exp(-2\zeta_j L) \right] \tag{19}$$

$$P_h = 16n^2 D_x \sum_{j=1}^{\infty} \frac{A_j J_1(\mu_j)\, \zeta_j}{B_j \mu_j} \exp(-\zeta_j L) \tag{20}$$

$$P_{hs} = \frac{8n^2 D_r}{\rho_0^2} \sum_{j=1}^{\infty} \frac{A_j J_1(\mu_j)\, \mu_j}{B_j \zeta_j} \left[1 + \frac{D_x \zeta_j}{\psi_2} + \left(1 - \frac{D_x \zeta_j}{\psi_2}\right) \exp(-2\zeta_j L) - 2\exp(-\zeta_j L) \right] \tag{21}$$

$$E_h = 8n^2 k \sum_{j=1}^{\infty} \frac{A_j J_1(\mu_j)}{B_j \mu_j \zeta_j} \left[1 + \frac{D_x \zeta_j}{\psi_2} + \left(1 - \frac{D_x \zeta_j}{\psi_2}\right) \exp(-2\zeta_j L) - 2\exp(-\zeta_j L) \right] \tag{22}$$

4. INVERSE PROBLEMS OF RADIATION DIFFUSION

Individual or combinations of thermal radiative properties can be used as measured values in the solution of inverse radiation diffusion problems for determination of the optical properties k, D_x, and D_r. However, the value of R_h is near unity and the most sensitive thermal radiative properties for changing of the optical properties are P_h, P_{hs}, and E_h.

In the determination of optical properties, the values n, $r_{h1(2)}$, and r_{hs} are considered to be known, $r_{h1} = r_{h2} = r_{hs} \equiv r_h$. The effective refractive index is calculated by Eq. (8). As shown in Ref. 9, the internal bihemispherical boundary reflectances have a weak influence on the thermal radiative properties when the inequalities given by Eq. (1) are fulfilled. They can be determined using the simple model of boundary reflection [8-10].

It must be noted that application of Eqs. (19)–(22) requires diffuse (isotropical) illumination of cylindrical sample face 1. However, it is easier to use the collimated incident radiation beam in practice. This difficulty may be bypassed [8, 10] if measurement of the angular dependence of the directional–hemispherical transmissivity for the collimated incident radiation beam is performed. On the basis of this measurement, the value u_n, which is equal to the ratio of the normal-hemispherical transmissivity to the bihemispherical one, can be obtained. This value does not depend on the thickness of the sample, so it can be obtained from measurements on a single sample of arbitrary thickness.

In this study, the normal-hemispherical transmissivities $P_n^{\parallel}$ were measured for a set of cylindrical samples of different thicknesses ($L = 4$–20 mm, $\rho_0 = 15$ mm), which were cut from a slab so that the axis of material structure symmetry was parallel to the axes of samples and perpendicular to the base of the slab.

As the value of $P_n^{\parallel}$ is not sensitive to changing of $D_{\perp} \equiv D_r$, measurements of $P_n^{\parallel}$ were supplemented with measurments of the normal-hemispherical transmissivity $P_n^{\perp}$ of another set of samples. These samples were cut from the same slab so that the axis of material structure symmetry was perpendicular to the axes of samples. When doing this, thin samples ($L/\rho_0 \ll 1$) of the same or similar thickness are used. In this case, the unidimensional expression for bihemispherical transmissivity can be applied:

$$P_h^{\perp}(L) = \frac{8n^2 D_{\perp} \zeta_{\perp} \exp(-\zeta_{\perp} L)}{(1 + D_{\perp}\zeta_{\perp}/\psi)^2 - (1 - D_{\perp}\zeta_{\perp}/\psi)^2 \exp(-2\zeta_{\perp} L)} \tag{23}$$

where $\zeta_{\perp} = (k/D_{\perp})^{1/2}$, and $\psi = 0.5(1 - r_h)/(1 + r_h)$.

The inverse problem of the radiation diffusion is formulated as a problem of a determination of such values as k, $D_{\parallel} \equiv D_x$, and $D_{\perp} \equiv D_r$, which give the minimum (as prescribed by the least-squares method) discrepancy of the calculated values of transmissivities from the measured ones.

5. RESULTS

Fibrous silica glass fiber thermal insulation with a density of 262 $kg \cdot m^{-3} (\Pi = 0.88)$ was considered as the material whose optical characterisitics were to be studied. During fabrication, the thermal insulation took a slight anisotropic structure due to some preferable orientation of fibers parallel to the slab base. The mean radius of fibers was about 1.5 μm; the mean length was about 70 μm.

The method of measurement and the experimental arrangement were described earlier [10, 11]. Figure 1 shows the obtained experimental data on normal-hemispherical transmissivities $P_n^{||}$ of a set of cylindrical samples of different thicknesses. Their axes coincided with the axis of material structure symmetry. The plotted theoretical curves correspond to the values $P_n^{||}$, calculated by Eq. (20) with the additional use of the expression $P_n^{||} = u_n P_h^{||}$. The data on u_n were obtained experimentally and its values were 0.806, 0.813, and 0.826 for $\lambda = 0.63$, 1.15, and 3.39 μm.

The data on k, $D_{||} = D_x$, and $D^{||} = D_r$ were obtained by the least-squares method to minimize the function defined by the difference between calculated and measured values of $P_n^{||}$ [3]. To obtain more reliable data on D_r, the measured values of $P_n^{\perp}$ were included in the fitting of k, D_x, and D_r. The obtained experimental data on $P_n^{\perp}$ are presented in Table I. As the

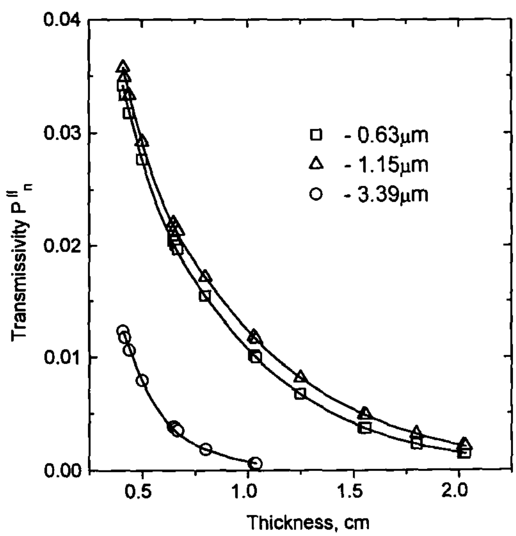

Fig. 1. Normal-hemispherical transmissivity $P_n^{||}$ of cylindrical samples of the anisotropic silica fiber thermal insulation (the discrete points represent experimental values and the curves represent calculated values).

Table I. Experimental Data on Normal-Hemispherical Transmissivities $P_n^{\perp}$

Thickness L (cm)	$P_n^{\perp}$		
	$\lambda = 0.63\ \mu m$	$\lambda = 1.15\ \mu m$	$\lambda = 3.39\ \mu m$
0.41	0.0495	0.0421	0.0164
0.41	0.0419	0.0429	0.0166
0.42	0.0404	0.0412	0.0157
0.42	0.0395	0.0407	0.0158
0.43	0.0397	0.0396	0.0150
0.43	0.0393	0.0393	0.0151

thicknesses of the samples were small, we used the unidimensional expression, Eq. (23), for $P_h^{\perp}$ calculations. The obtained data on optical properties and the factor of optical anisotropy are given in Table II.

Comparison of these data with the results of other authors is not possible, as experimental study of optical properties of anisotropic fiber thermal insulation by others has not yet been made. The numerical calculations by Lee [5–7] give only qualitative information on the influence of fiber orientations on a scattering phase function for a medium with monosize fibers. The optical properties of some thermal insulation in those studies were not calculated.

The thermal radiative properties (bihemispherical reflectivity R_h and hemispherical absorptivity E_h) of an infinite plane layer of thermal insulation studied by us as a function of thickness were calculated on the basis of obtained optical properties. Some representative results are shown in Figs. 2 and 3. The anisotropy of the radiation diffusion coefficient has a different influence on R_h and E_h. The reflectivity $R_h^{\perp}$ of the plane layer whose normal is perpendicular to the axis of material structure symmetry is less than $R_h^{\parallel}$ of the layer of other orientation of the axis of structural symmetry, i.e., $R_h^{\perp} < R_h^{\parallel}$, and vice versa, for absorptivities, i.e., $E_h^{\perp} > E_h^{\parallel}$. It can be

Table II. Optical Properties of Anisotropic Silica Fiber Thermal Insulation

Wavelength λ (μm)	Effective absorption coefficient, 10^3 k(cm^{-1})	Radiation diffusion coefficient		Optical anisotropy factor, $a = D_{\parallel}/D_{\perp}$
		10^3 $D_{\parallel}$ (cm)	10^3 $D_{\perp}$ (cm)	
0.63	2.6 ± 1.3	2.52 ± 0.08	3.17 ± 0.15	0.80 ± 0.05
1.15	0.80 ± 0.15	2.58 ± 0.04	3.03 ± 0.04	0.85 ± 0.02
3.39	36 ± 20	1.59 ± 0.040	1.92 ± 0.40	0.82 ± 0.12

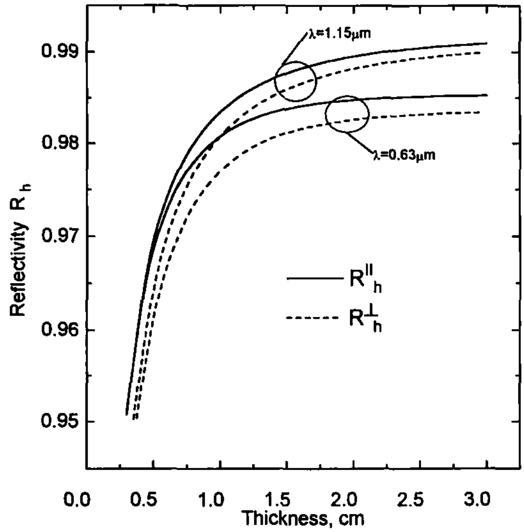

Fig. 2. Bihemispherical spectral reflectivity R_h of a plane layer of anisotropic silica fiber thermal insulation of different orientations.

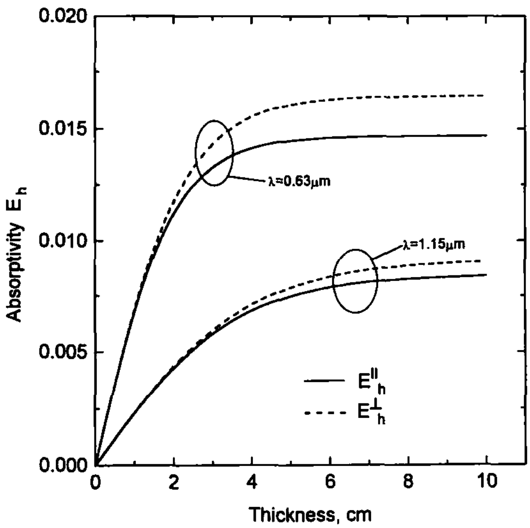

Fig. 3. Hemispherical spectral absorptivity E_h of a plane layer of anisotropic silica fiber thermal insulation.

explained by applying the expressions for R_h and E_h obtained in Ref. 9 to the optically thin relative to attenuation ($\zeta L \ll 1$) and the optically thick ($\zeta L \gg 1$) layers. For the optically thin layer, the value $1 - R_h$ is proportional to D, and E_h does not depend on D. As $D_{\parallel} < D_{\perp}$, then $R_h^{\parallel}(L) > R_h^{\perp}(L)$ and the curve $E_h^{\parallel}(L)$ coincides with the curve $E_h^{\perp}(L)$ at small values of L.

For the optically thick layer, R_h and E_h do not depend on thickness and they are related to each other by the expression

$$1 - R_h = E_h = 4n^2(kD)^{1/2} \tag{24}$$

It follows that the curves $R_h(L)$ and $E_h(L)$ approach constant values at large L. In this case, the inequalities $R_h^{\parallel}(L) > R_h^{\perp}(L)$ and $E_h^{\parallel}(L) < E_h^{\perp}(L)$ apply.

As the inequality $D_{\parallel} < D_{\perp}$ does not change whith wavelength, the above correlations between $R_h^{\parallel}$ and $R_h^{\perp}$, and between $E_h^{\parallel}$ and $E_h^{\perp}$ remain valid with changing wavelength.

6. CONCLUSION

The method developed for the determination of optical properties of scattering semitransparent anisotropic materials on the basis of the diffusion model allows one to describe the radiation transfer even when RTE is not valid. The optical properties of such materials have been described in terms of three coefficients only: the effective absorption coefficient k and the axial D_x and radial D_r components of the tensor of the radiation diffusion.

ACKNOWLEDGMENT

This research was supported by Russian Foundation for Fundamental Research Grant N94-02-06570.

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